Field Scale Fertilizer Recommendations and Spatial Variability

R.Gary Kachanoski*,[1] and Gordon L. Fairchild[2]

[1]Associate professor and [2]Research associate, Dept. of Land Resource Science, University of Guelph, Guelph, Ontario, Canada. N1G 2W1.

Abstract

Fertilizer is usually applied at a single constant rate across a field. However, soil fertility may vary considerably within a field. Soil test calibrations (ie recommended fertilizer versus soil test) are usually obtained from sites with low spatial variability of soil test values (ex. small plots). These calibrations are then assumed to be valid for large fields with variable soil fertility. The effects of variable soil fertility on the relationship between average crop yield response, average soil test, and fertilizer applied evenly to a field has not been examined. This paper develops stochastic equations to describe the average field yield gain from the application of a constant rate of fertilizer, in fields with variable soil fertility. The equations are solved numerically for the case of nitrogen fertilizer on corn.

The results indicate that since the relationships between yield response, soil test, and applied fertilizer are non-linear, a single soil test calibration cannot exist for fields with different spatial variability.Soil test calibrations obtained from sites with low variability of soil test values (ex. small plots) will not hold for sites with higher variability (ex. farm fields). In addition, calibrations obtained from sites with low variability of soil test values (ex. small plots) will under predict the optimum fertilizer rate for maximum economic yield for sites with high variability of soil test. The results do not invalidate soil test calibration relationships.In fact, because of the spatial scaling problem, it more important than ever to have accurate calibrations. The challenge is to combine these calibrations with additional knowledge about the spatial distribution and field scale variability of soil test values.

Introduction

It is well known that many fields have significant spatial variability of soil fertility. This is the basis behind the development of technology to spatially vary the application rate of fertilizer within a field. However, a majority of fields still have a single rate of fertilizer applied evenly across the field. Soil test calibration relationships (ie recommended fertilizer rate versus soil test values) are used to obtain the recommended fertilizer rate from a soil test on a composite soil sample from the field.

It is also well known that the relationship between yield response, applied fertilizer, and soil fertility levels (soil test) is highly non-linear. Quadratic equations and other non-linear yield response models are often used. Unfortunately, there are significant problems with estimating spatial averages of non-linear relationships.The following example illustrates the spatial scaling problem.

Suppose you have 2 fields (Fields# I, and II) as given in Table 1. Both fields have the same average soil test value (x=23), but three areas within each field have different soil test values. Note that the only difference between the 2 fields is the relative proportion of each of the three areas . The average soil test, x is obtained from the sum of the value of the soil test in each of the different areas multiplied by the proportion of the field occupied by each area. This average soil test would be equal to the average soil test from a composite soil sample taken from the field.

It is assummed that a relationship between maximum yield gain from applied fertilizer, $\Delta Y_m(x)$ as a function of soil test, x, exists and is known.It is assumed that the critical soil test value (the value were yield response to fertilizer is zero) is x = 30. Thus, $\Delta Y_m(x) = 0$ for $x \geq 30$. The values of the yield increases for soil test values x=20 and x=10 are assumed to be 1000 Kg/ha and 3000 Kg/ha, respectively. The average maximum yield gain for each field, ΔY_m is calculated from the maximum yield gain in each of the 3 areas with different soil test values multiplied by the proportion of the field occupied by each area. This average yield gain would be the yield increase obtained from the entire field (machine harvested yield).

The average maximum yield increase possible from applied fertilizer is 900 Kg/ha in field(I) compared to 1500 Kg/ha in Field(II), even though both fields have the same average soil test value and the same yield response for a given soil test level within the fields. The different average yield responses of the two fields is caused by the non-linear relationship between soil test and crop yield response.If yield response decreased linearly with increasing soil test values then both fields would have the same average yield increase. The sharp non-linear change in yield response near the critical value is the main cause of the spatial averaging problem. Since many (all?) nutrient calibrations have similar yield reponse relationships, the problem is widespread.

Objective

The objective of this study was to develop equations which describe the average field yield increase from fertilizer applied evenly to the whole field, in a field with a probability distribution of soil test values (ie variable soil fertility). The derivation of the equations are given for a generalized yield response to applied ferrtilizer, and then are solved for the specific case of corn response applied N fertilizer in Ontario, Canada.The equations are given in the Appendix:Mathematical Theory at the end of the paper. A verbal description of the theory is given in the following paragraphs.

Methodology

The procedure used to evaluate the average yield response to applied fertilizer in a field with spatially varying soil test was as follows,

1.Define the relationship between yield increase (ie. the yield with fertilizer minus the yield without fertilizer), soil test, and rate of fertilizer applied, for the case of zero spatial variability. The relationship must give actual yield increases and not just relative yield increases. The equation does not have to be 100% accurate, since the purpose is to determine how the relationship (equation) changes as the variability of the soil test within the field changes.

2. Assume a field has a particular average soil test and spatial variability of soil test. For a particular fertilizer application rate, calculate the yield increase expected for any soil test value using the relationship defined in step 1.. Multiply the calculated yield increase by the proportion of the field which would have this yield increase. This proportion is equal to the probability of getting the given soil test value, which depends on the average and variability of the soil test in the field.

3. Repeat the calculation in step 1, for all possible soil test values in the field (keeping the applied fertilizer rate constant).The possible values are defined by the assumed average and variability of soil test in the field. Calculate the field average yield increase by adding up all the different proportions of the field with their respective yield increases. This gives the field average yield increase for a single evenly applied fetilizer rate, in a field with a particular average and variability of soil test.

4. Repeat steps 2 and 3 for different rates of applied fertilizer, keeping the average soil test and variability the same. Plot average yield increase as a function of applied fertilizer rate and determine the rate which maximizes the field average economic return. This gives the recommended rate of fertilizer application for a field with a particular average and variability of soil test.

5. Repeat steps 2 through 4 for different average soil test values in combination with different levels of variability at each average soil test.

6. Plot recommended fertilizer rate as a function of average soil test for field with different amounts of variability

Results and Discussion

Figure 1 shows the predicted corn yield increase (ie yield with N fertilizer minus yield without N fertilizer) for a field as a function of N fertilizer applied when the N soil test (0-60cm) at planting time was 60 Kg NO_3-N/ha. The field average yield response to applied N fertilizer increased significantly as the variability of the N soil test increased. The maximum yield increase when there was no variability of soil test (ie the coefficient of variability,CV=0) was 1200 Kg grain/ha. With a soil test CV=50% a yield increase of 3400 Kg grain/ha is predicted. The maximum economic N fertilizer rate,MERN was 110 KgN/ha for CV=0, and increased to 150 KgN/ha for CV=50%. Similar data can be generated for any N soil test level.

The relationship between maximum economic rate of fertilizer N (MERN) and the N soil test (NTEST) is given in Figure 2, for different CV's of soil test. The curve for CV=0 is the calibration curve for fields with zero variability. The calibration for CV=0 is similar to calibration curves reported by other researchers in Humid North-Central Areas. As the CV of a field increases the maximum economic rate of N fertilizer recommended also increases.The largest increase in recommended fertilizer rate occurs at the critical soil test level .

The maximum difference in yield increase from fertilizer application,between variable and non-variable fields occurs in the mid-range of the NTEST values(Figure 3). For NTEST=40 and N fertilizer applied at a rate of 120 KgN/ha, the yield increase was 2500 Kg/ha for CV=0, compared to a yield increase of 6500 Kg/ha for CV=50%. Note that this does not mean that the variable field is better because it responded more to applied fertilizer. The check yield (not shown) is significantly lower in the variable field compared to the field with no variability. It is the increased probability of lower check yields in variable fields, that causes the increased probability of a larger response to applied fertilizer.

Although not given in this paper, similar equations have been solved for the influence of spatial variability of soil test on phosphorus and potassium fertilizer recommendations. The recommended K fertilizer rate as a function of both average soil test and spatial variability of soil test is given in Figure 4. The effects of the potassium soil test variability were similar to the previous results for nitrogen.

In summary, the major implications of the spatial scaling problem can be stated as follows.

Since the relationships between yield response, soil test, and applied fertilizer are non-linear, a single calibration (recommended fertilizer versus soil test) cannot exist for fields with different spatial variability.

Calibrations (recommended fertilizer versus soil test) obtained from sites with low variability of soil test values (ex. small plots) will not hold for sites with higher variability (ex. farm fields).

Calibrations (recommended fertilizer versus soil test) obtained from sites with low variability of soil test values (ex. small plots) will under predict the optimum fertilizer rate for maximum economic yield for sites with high variability of soil test.

The previous statements, if true, are rather disturbing, since a majority of fertilizer recommendations from soil tests are made from a composite soil sample from a field and a calibration relationship obtained from research plots selected for uniformity (ie low spatial variability of soil test). The results may also help explain why many farmers and fertilizer dealers insist they get an economical increase in yield with fertilizer application rates higher than those predicted by such calibration relationships. If they have a variable field, the theory presented here suggests they will get economic yield increases with higher rates. This does not invalidate the calibration relationship. It just means that we have to utilize the calibration relationship in a different manner.In fact, because of the spatial scaling problem it more important than ever to have accurate calibration relationships between soil test, yield response, and applied fertilizer. The challenge is to combine these calibrations with additional knowledge about the spatial distribution and field scale variability of soil test values.

APPENDIX : Mathematical Theory

The spatial average of soil test in the previous example from Table 1 was calculated from

$$\bar{x} = P_1 * x_1 + P_2 * x_2 + P_3 * x_3 \qquad ; P_1 + P_2 + P_3 = 1 \tag{1}$$

where P_1, P_2, P_3,= the proportion of areas 1, 2, and 3 respectively, and x_1, x_2, x_3= the soil test values in areas 1, 2, and 3 respectively. For fields with many different areas, equation(2) can be given by,

$$\bar{x} = \sum_{n=1}^{N} P_n\, x_n \qquad ; \sum_{n=1}^{N} P_n = 1 \tag{2}$$

where P_n= proportion of the field occupied by the n^{th} area, and x_n= the soil test value in the n^{th} area, n=1, 2,...,N .The proportion of a field occupied by a particular area can also be viewed as the probability that a single soil sample would be taken in that area. Thus, P_1 can be viewed as the total probability that a soil sample would have the soil test value x_1 (etc.). Most fields have a continuum of possible soil test values(ie. $N \to \infty$), which must be described by a probabilty density function. Thus, the average soil test value for fields is given by an integration rather than a summation,

$$\bar{x} = \int_o^\infty p(x)\; x\; dx \qquad ; \int_o^\infty p(x)\; dx = 1 \tag{3}$$

where p(x)= the probability density function of x. Equation(3) is identical to equation(1) except that you have a continuum of possible soil test value in the field , rather than just 3 possible values.

The previous example was for the case where the amount of fertilizer applied on the field was enough to make nutrients non-limiting for crop growth, even for the areas with the lowest soil test value.However, the same scaling problem exists for any single rate of fertilizer applied evenly to a field. We start with the assumption that <u>when soil test does not vary</u>, there is a definable relationship between yield gain, soil test, and applied fertilizer, which can be given as

$$\Delta Y_{N,x} = f\,(\,N\,,\,x\,) \tag{4}$$

where $\Delta Y_{N,x}$= yield gain(over check) from applied fertilizer, N_a=rate of fertilizer applied, x= soil test value. Equation (4) can also be rearranged to give the inverse relationship

$$x = g\,(\,\Delta Y_{N,x}\,,\,N\,) \tag{5}$$

For a field with variable soil test, the average yield gain from evenly applied fertilizer can be given by

$$\overline{\Delta Y_N} = \int_o^{\infty} p(\Delta Y_{N,x})\ \Delta Y_{N,x}\ d(\Delta Y_{N,x}) \qquad ; \int_o^{\infty} p(\Delta Y_{N,x})\ d(\Delta Y_{N,x}) = 1 \tag{6}$$

where ΔY_N= the field average yield gain, and $p(\Delta Y_{N,x})$= the probability density function of yield gain for the field. Since yield gain and soil test have a definable relationship once the fertilizer rate is given, it is possible to relate their probability density functions through

$$p(\Delta Y_{N,x}\,) = p(x)\ \left|\frac{dx}{d(\Delta Y_{N,x}\,)}\right| \tag{7}$$

The probability density function of soil test is assumed to follow a lognormal distribution given by

$$p(x) = \frac{1}{x\sigma_x\sqrt{2\pi}}\,e^{-\frac{1}{2}\left[\frac{\ln(x)-\mu_x}{\sigma_x}\right]^2} \tag{8}$$

where σ_x and μ_x are the standard deviation and average value of ln(x), respectively. Substitution of equations (8), (7), and (5) into equation (6) gives

$$\overline{\Delta Y_N} = \int_0^\infty \left[\frac{1}{g(\Delta Y_{N,x},N)\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(g(\Delta Y_{N,x},N))-\mu_x}{\sigma_x}\right]^2} \right] \left| \frac{d(g(\Delta Y_{N,x},N))}{d(\Delta Y_{N,x})} \right| \Delta Y_{N,x}\, d(\Delta Y_{N,x}) \tag{9}$$

Equation (9) gives the average yield gain from an even application of fertilizer onto a field with a variable soil test.The mean and variance of the soil test are described by μ_x and σ_x . Equation (9) must be solved numerically .

Application to N Fertilizer Resonse

The relationship between yield gain and a applied rate is assumed to be given by

$$\Delta Y_N = B*N - C*N^2 \qquad ; N \leq N_{MAX} \tag{10}$$

$$\Delta Y_N = B*N_{MAX}\ C*N_{MAX}{}^2 \qquad ; N \geq N_{MAX} \tag{11}$$

$$N_{MAX} = \frac{B}{2C} \tag{12}$$

$$N_R = N_{MAX} - \frac{R}{2C} \tag{13}$$

where B and C are regression constants, N_{MAX}= the N fertilizer rate at the start of the maximum yield (ie $d(\Delta Y)/dN = 0$), R= the ratio of the price per Kg of fertilizer/ price per Kg grain, and N_R= recommended rate of fertilizer N for a particular price ratio R. The equation for N_R is obtained by taking the first derivative of equation (10), setting it equal to the price ratio and solving for N_R. We assume a calibration between N_R and soil test x exists for the case of zero spatial variability of x (ie $\sigma_x^2 = 0$), and can be given by

$$N_R = N_{MAX} - \beta_1*(x-\beta_0) \qquad ; x_{\min} < x \leq x_c \tag{14}$$

$$N_R = 0 \qquad ; x \geq x_c \tag{15}$$

where x_c = the critical soil test value for zero fertilizer recommendation, β_0 and β_1 are calibration constants, and x_{min} is the minimum soil test value that equation(14) is valid for. Since

$N_R < N_{MAX}$, the value of $x_{min} > \beta_0$. Given a value of N_{MAX} , x_{min} is the soil test associated with the maximum possible N_R. For Ontario the maximum value of $N_R \approx 0.975 N_{MAX}$. Substitution of equations (11) through (15) into equation (10) gives simple relationship for soil test , yield gain, and applied fertilizer N (ie.the functions f(N,x), and g ($\Delta Y_{N,x}$,N) in equations (4) and (5))

$$\Delta Y_{N,x} = f(N, x) = \frac{\alpha(N)}{x - \beta_o} \qquad ; x \geq x_{\min} \tag{16}$$

$$x = g(N, x) = \frac{\alpha(N)}{\Delta Y_{N,x}} + \beta_0 \qquad ; x \geq x_{\min} \tag{17}$$

$$\Delta Y_{N,x} = \Delta Y_{MAX} \qquad ; x \leq x_{\min} \tag{18}$$

where

$$\alpha(N) = \left[\frac{R\, N_{MAX}}{\beta_1}\right] N - \left[\frac{R}{2\beta_1}\right] N^2 \tag{19}$$

$$x_{\min} = \frac{\alpha(N_{MAX})}{\Delta Y_{MAX}} + \beta_0 \tag{20}$$

Equations (16) through (20) can be substituted into equation (9) to calculate the average yield increase as a function of N fertilizer applied evenly to a field. Parameters needed to solve the equations are; β_0 and β_1 from N calibration trials, N_{MAX} which can be set for a particular field, and μ_x and σ^2_x which describe the mean and spatial variability of the soil test. The equations were solved for corn response in Ontario conditions with N_{MAX}=175 KgN/ha, β_0=25.6 Kg NO_3-N/ha, and β_1=1.95.

Table 1: Example of spatial scaling problem.

	Area #	Percent of Field	Soil Test	Maximum Crop Response to Fertilizer
	1	50%	30 (high)	0 kg/ha
Field 1	2	30%	20 (medium)	1000 kg/ha
	2	20%	10 (low)	3000 kg/ha
		100%	average = 23	Average 900 kg/ha
Average soil test = (0.5 x 30) + (0.3x20) + (0.2 x 10) = 23				
Average yield gain = (0.5 x 0) + (0.3 x 1000) + (0.2 x 3000) = 900 kg/ha				
Field II	Area #	Percent of Field	Soil Test	Crop Response to Fertilizer
	1	20%	60 (high)	0 kg/ha
	2	30%	20 (medium)	1000 kg/ha
	3	50%	10 (low)	3000 kg/ha
		100%	Average = 23	Average = 1500 kg/ha
Average soil test = (0.2 x 60) + (0.3 x 20) + (0.5 x 10) = 23				
average yield gain = (0.2 x 0) + (0.3 x 20) + (0.5 x 3000) = 1800 kg/ha				

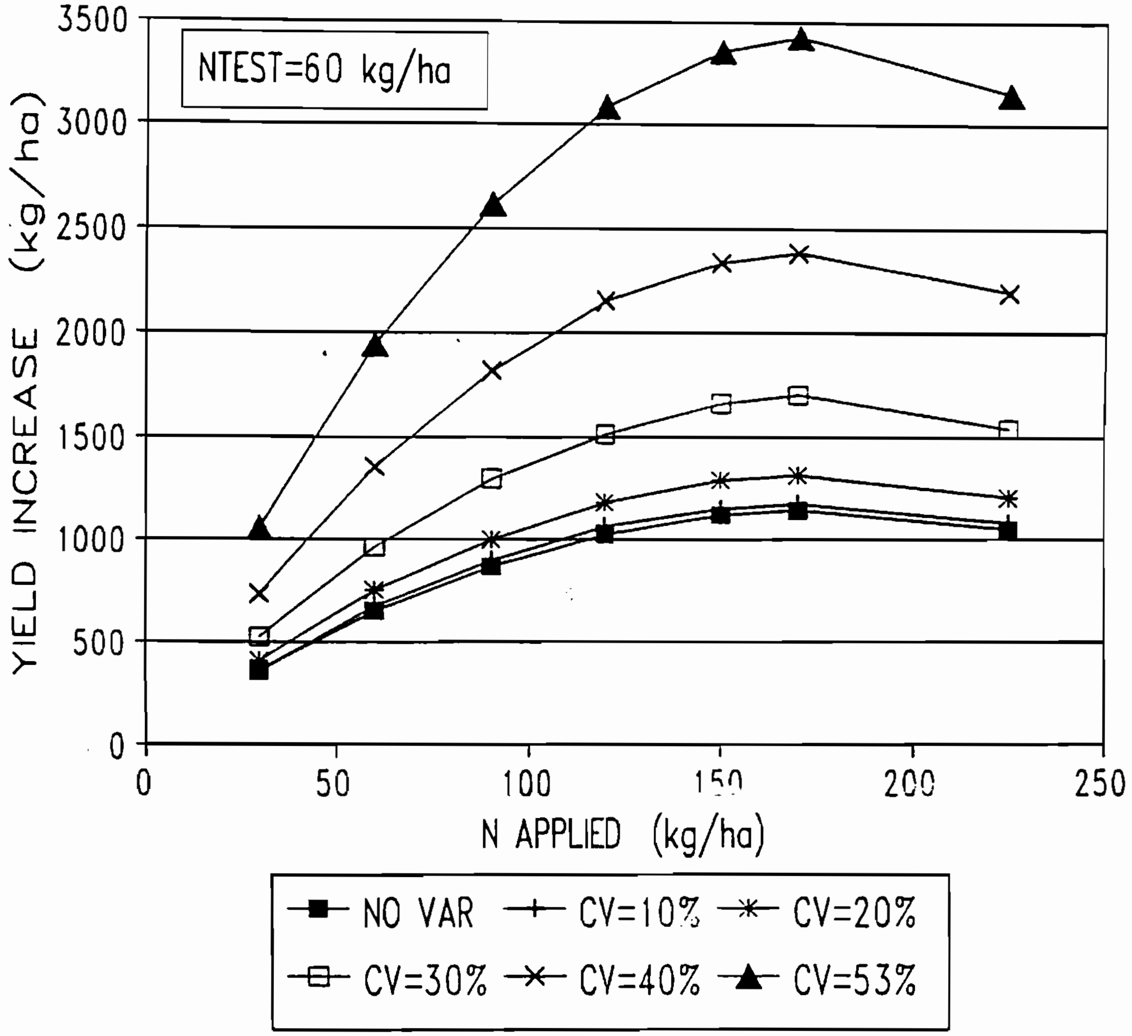

Fig. 1. Influence of NTEST variability on corn yield increase, at R=5, and NTEST= 60 kg/ha. (CV = coefficient of variability)

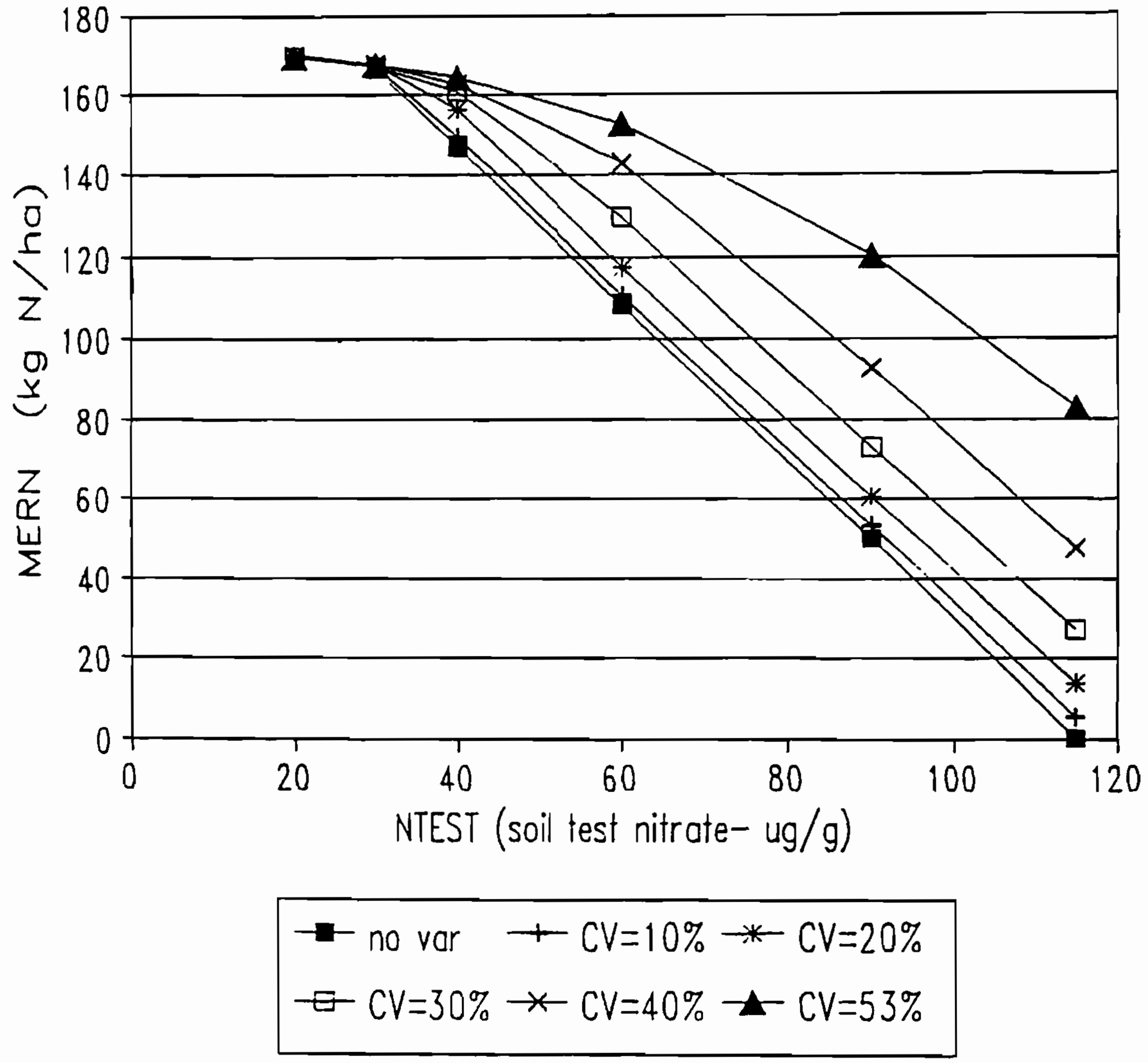

Fig. 2. Influence of NTEST variability on fertilizer N rate at R=5, (CV = coefficient of variability)

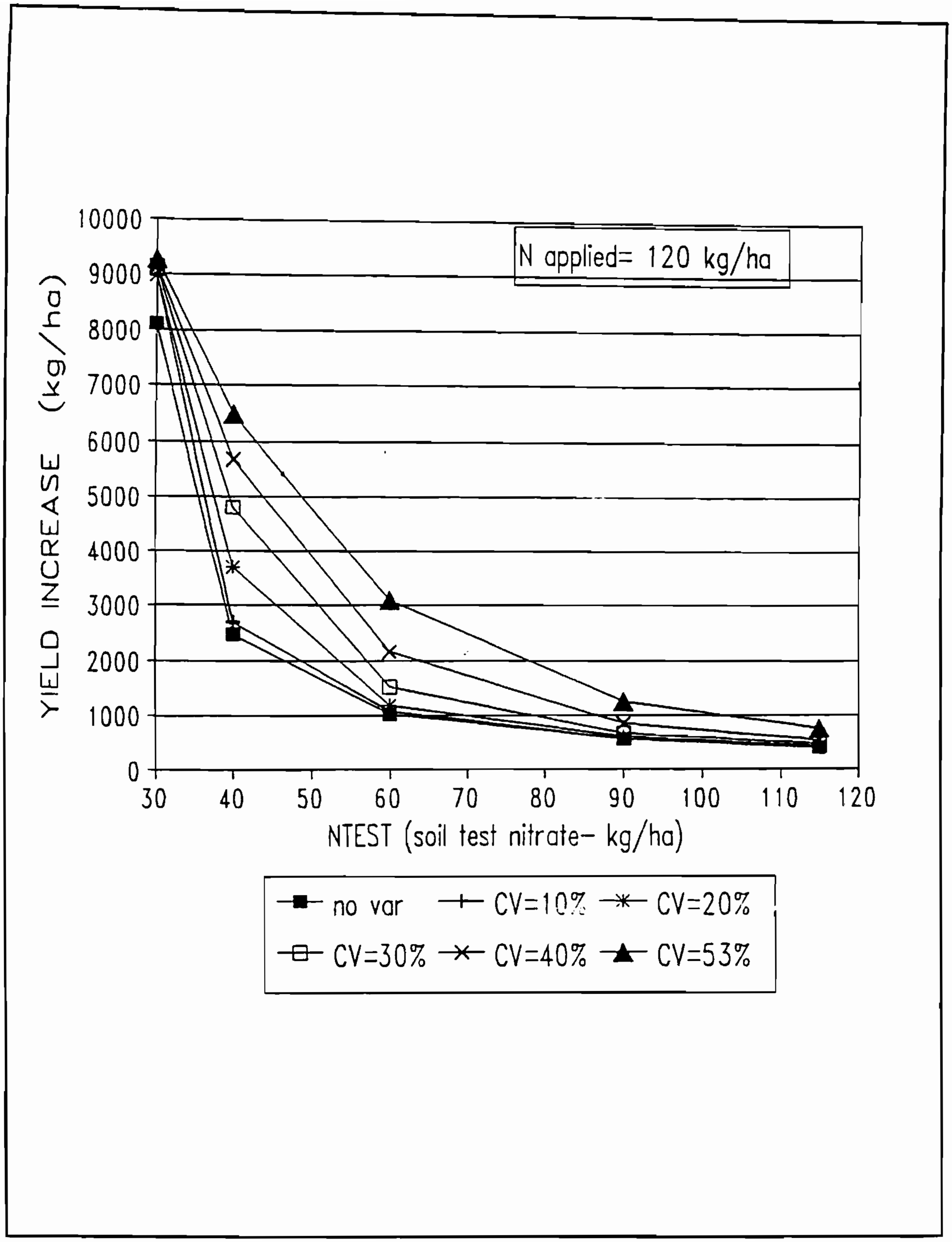

Fig. 3. Influence of NTEST variability on corn yield increase, at R=5, and N applied =120 kg/ha. (CV = coefficient of variability)

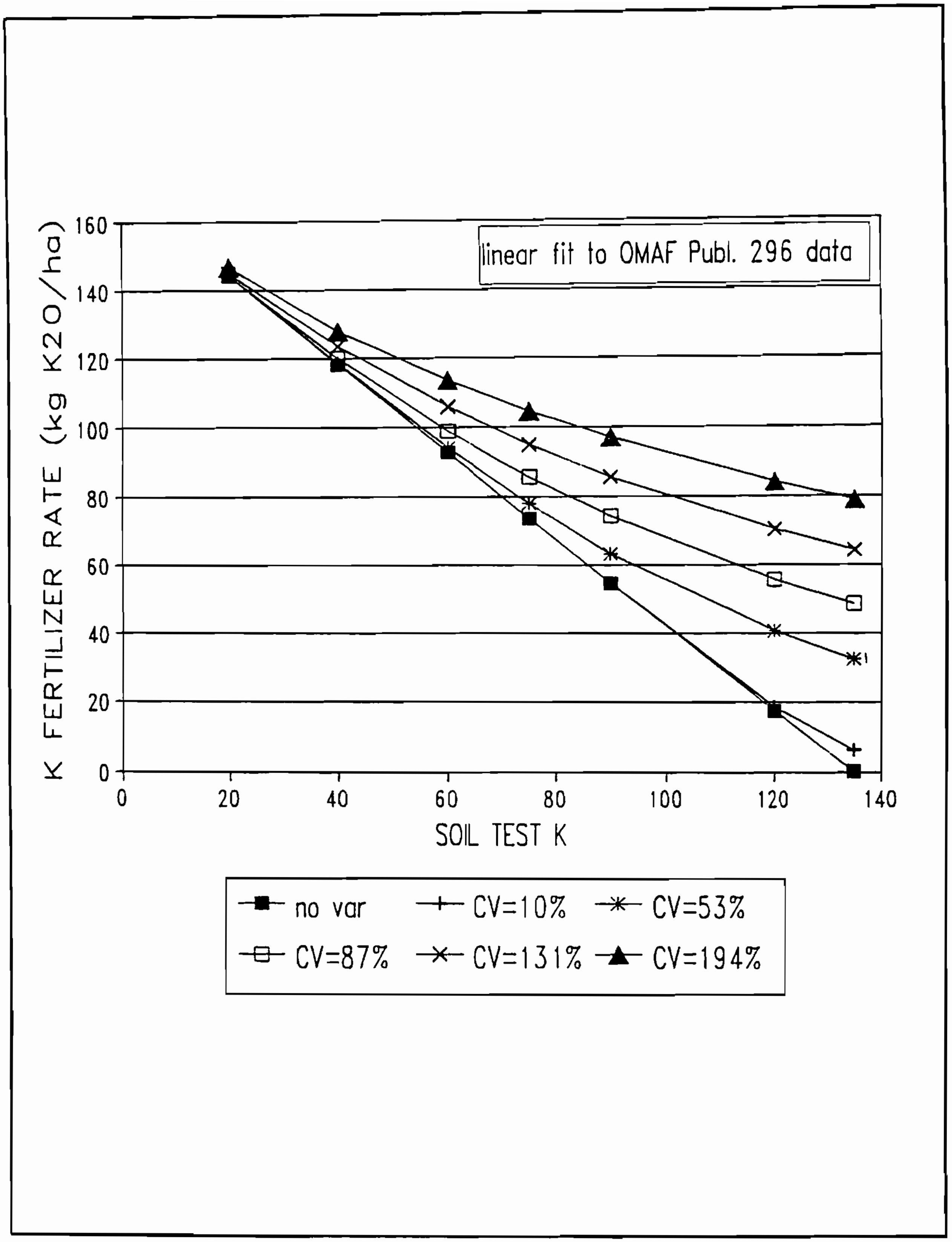

Fig. 4. Influence of soil test K variability on K fertilizer rate, utilizing a linear fit to the OMAF Publ. 296 K fertilizer rate recommendation data.

PROCEEDINGS OF THE TWENTY-THIRD
NORTH CENTRAL EXTENSION - INDUSTRY
SOIL FERTILITY CONFERENCE

October 27-28, 1993, Holiday Inn St. Louis Airport

Bridgeton, Missouri

Volume 9

Program Chairman and Editor:

Dr. Lloyd Murdock
University of Kentucky
Research and Education Center
P.O. Box 469
Princeton, KY 42445